647510 **MSC2MatC203x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
August/September -2020 [NEW COURSE]
Topology – 2 (CORE)

Time: 2:00 Hours **Marks: 56**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any four of the following questions.

Q.1 Answer any seven from the following. (14)

(1) Define topological space with example.
(2) Construct any two topologies on $X = \{1,2,3,4\}$.
(3) Find set of limit points of a subset $A = (2,4] \cup \{0\}$ of R.
(4) Write down any two properties of a closure of a set.
(5) Define compact space with example.
(6) Give definition of a completely regular space.
(7) For a subset $A =$ of R, find closure of A.
(8) Give an example of a Cauchy sequence which is not convergent in a metric space.
(9) Justify that every Hausdorff space need not be regular.
(10) Discuss the smallest and largest topology on a non-empty set X.

Q.2 Attempt any two. (14)

(1) Define T_1 and T_2 - spaces and prove that a subspace of T_1 space is T_1.
(2) Show that the product of two Hausdorff space is Hausdorff.
(3) Define complete metric space and show that the complex plane C is a complete metric space.

Q.3 Answer the following. (14)

(1) Define convergent sequence with example and show that every convergent sequence is a Cauchy sequence.
(2) Let X be a complete metric space and Y be a subspace of X. Then prove that Y is complete if and only if Y is closed.

OR

Q.3 Answer the following. (14)

(1) Prove that every compact space is a limit point compact space.
(2) Discuss compactness property of R in detail.

Q.4 Attempt any two. (14)

(1) Define normal spaces and prove that a space X is T_1 if and only if every singleton is closed in X.
(2) Show that the set of rational numbers Q is not locally compact space.
(3) Prove that every closed subspace of a compact space is compact.

Q.5 Attempt any two. (14)

(1) Show that a product of regular spaces is regular.
(2) Define locally compact space. Give an example of a locally compact space which is not compact.
(3) Let X be a locally compact Hausdorff and let A be a subspace of X. If A is closed in X or open in X, then prove that A is locally compact.
(4) Show that if X is normal, then every pair of disjoint closed sets have neighborhoods whose closures are disjoint.

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